Intersect, Vol 6, No 1 (2013)

Mathematical Modeling: The Whewellian Influence on Maxwell's Geometrical Analogies

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With the advent of quantum mechanics came the Copenhagen interpretation that scientists can only speak about their experimental findings in terms of probabilities. This school of thought is contingent upon the idea that statistical reasoning can be accurately applied to physical theories. Influenced by the likes of astronomer Adolphe Quetelet and historian Henry Buckle, James Clerk Maxwell gets, "the credit for first introducing the explicit consideration of probability distributions into physics" (Harman, 1990; Porter 1988). At the age of fifteen, Maxwell authored his first scientific paper, "Paper on the Description of Oval Curves", which examines a mechanical description of ovals (Harman, 1990). His nascent geometrical interests would eventually flower into a full-fledged component of his science: relations by analogy. This method, which Maxwell explains in his paper "On Faraday's Lines of Force", "combines the advantages" and "gets rid of the disadvantages both of premature physical theories and technical mathematical formulæ," relates an abstract property to a more concrete notion in order to make a topic of inquiry more comprehensible. Maxwell dubs this approach "the method of Physical Analogy" and points to "the use of lines in mechanics to represent forces and velocities" as an example illustrative of this method (Harman, 1990). Though his early fascinations with geometry underscore his proclivity to turn to mathematics for answers, the young Maxwell did not stumble upon this important aspect of his method without aid. His master at Trinity College, Cambridge – William Whewell – influenced the development of this method of analogy by emphasizing mathematical rigor and promoting the principle of superinduction.

Whewell's advocacy for a structured study of mathematics supplied Maxwell with the geometrical foundations later extant in Maxell's analogies. In the treatise, "On Mathematical Reasoning," Whewell describes how "we cannot conceive or perceive objects at all, except as existing in space; we cannot contemplate them geometrically, without assuming those properties… which are the basis of geometry" (Whewell, 1837). This *a priori* justification for geometry highlights its central role in Whewell's epistemology and contributes to his notion that one can first mathematically spell out fundamental principles of doctrine prior to confirming them through experimentation (Harman, 1998). Whewell's belief in the importance of a strong mathematical education manifested itself in his zealous introduction of the Mathematical Tripos, an intensive examination system which determined the allocation of university fellowships. Whewell was a staunch champion for "the case for the preeminent role of mathematics (and, in particular, geometry) in Cambridge" (Gascoigne, 1984). Into the midst of this revamped curriculum stepped Maxwell's mentors from the Edinburgh Academy, who wrote to Whewell describing Maxwell as a budding genius who "required the discipline of systematic and ordered mathematical education" (Harman, 1998). After winning a fellowship to Trinity, Maxwell received an education with a profound stress on mathematics. His appropriation of Whewell's notion, that math possesses explanatory power prior to experimentation, is evident in his aptly-named "Mathematical Theory of the Composition of Colours, verified by quantitative experiments." The work of his contemporaries concerning color theory was highly theoretical, but Maxwell's title exemplifies the idea that math should be applied to model physical phenomena. In a gesture of approbation, Whewell nominated this paper for the Royal Medal of the Royal Society (Harman, 1998).

Whewell also shaped Maxwell's method of analogy through the principle of superinduction. Since the time of Bacon, Whewell contends that the business of induction is primarily concerned with "the *interpretation* of facts… the superinduction of an idea upon the facts by an interpreting mind" (Whewell, 1837). This method of reasoning impacted a young Maxwell who, in a personal correspondence with Richard Litchfield, confided that he enjoyed "grinding out 'appropriate ideas' as Whewell calls them" and "knocking them against all the facts" (Harman, 1990). As a disciple of this inductive mode of reasoning, Maxwell appropriated it to create his method of analogy. According to Maxwell's article "Analogies in Nature", whenever men see a relation they know well, they can use it to describe a less well-known one because "although pairs of things may differ widely from each other, the *relation* in the one pair may be the same as that in the other" and "in a scientific point of view, the *relation* is the most important thing to know" (Harman, 1990). Thus, Maxwell used Whewell's superinduction to collide his ideas with facts and come up with the "relations" he deemed so crucial to his projects.

The notion of placing value upon geometry's explanatory power, in conjunction with that of factual interpretation, combine to form Maxwell's method of geometrical analogy. He displays this method in his first foray into electromagnetism through his article, "Theory of the Motion of an Imponderable and Incompressible Fluid" which served as a draft for, "On Faraday's Lines of Force". Maxwell writes that he supposes a substance that "does not possess any of the properties of ordinary fluids except that of freedom of motion" and "is not even a hypothetical fluid" but rather, is "simply a collection of imaginary properties, which may be employed for establishing certain theorems in pure mathematics in a way more intelligible to ordinary minds" (Harman, 1990). The term "fluid" is Maxwell's way of analogizing electromagnetic fields in order to leverage greater explanatory power. This characteristic would become a hallmark of his work, appearing throughout *The Theory of Molecules* itself to describe even the smallest details, such as how "each molecule… bears impressed on it the stamp of a metric system as distinctly as does the metre of the Archives of Paris, or the double royal cubit of the Temple of Karnac" (Maxwell, 1873). He employs rather ornate metaphors here to simply state that molecules possess an equality to those of their kind.

These last examples reveal a sliver of the extent to which geometrical analogy factored into Maxwell's method of electromagnetism. Whewell's ideas left an indelible mark on Maxwell's mode of inquiry, as evidenced by Maxwell's reception of the dominant explanatory power of mathematics and of interpreting facts through superinduction. The resulting geometrical analogy – the quest for a *relation* between ideas – is part of what eventually led Maxwell to see the connections between humans and atoms, to appropriate statistical probability by reaching across the boundaries of discipline, and to shape the field of modern physics.

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