Intersect, Vol 18, No 1 (2024)

Urban Population Scaling and CO_2 Emissions

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Abstract

As the world continues struggling with climate change, it is important to better understand the causes, trends, and beneficial steps that can be taken for improvement. In this paper, we investigate the interplay between a country's population distribution across urban centers and its environmental footprint. Specifically, we propose a new measure –which we refer to as the "*Urban Scaling Factor*"– connecting a country's per-capita CO_2 emissions to its level of urbanization. The Urban Scaling Factor is based on a combination of Zipf's law and the Urban Scaling Hypothesis. With this new factor, we propose a model to determine whether a country would benefit from a shift in its population, either towards increased urbanization or dispersion. Additionally, the model predicts the hypothetical maximum reduction in emissions corresponding to these population shifts.

Subject Classifications: Environmental Science, Earth Science, Data Analytics.
Keywords: CO_2 Emissions, Environmental Kuznets Curve, Zipf's Law

I. Introduction

The relationship between a country's population and its level of carbon dioxide (CO_2) emissions is multifaceted (West, 2018). On one hand, as populations grow, so does the demand for energy, the consumption of goods, and the need for services, which can lead to higher CO_2 emissions. On the other hand, a larger population can also lead to economies of scale, technological innovations, and increased awareness of environmental issues, which can result in lower CO_2 emissions per capita. In this paper, we use a data-driven approach to shed new light on the complex relationship between CO_2 emissions and population size. Our objective is to offer an environmental perspective that can guide governments and policy makers in effectively addressing the tension between population growth and sustainable development.

While population growth is not the only factor that affects CO_2 emissions, it is one of the most important. According to the World Bank (2023), global CO_2 emissions have increased by over 50% since 1990 alongside a 30% global population increase. However, the effects of population growth on emissions are not uniform across countries. While some countries with high population growth rates have managed to reduce their CO_2 emissions per-capita through policies and investments in renewable energy, energy efficiency, and public transportation, others have seen their emissions skyrocket during the same period (UN Environmental Programme, 2023).

One possible explanation for this phenomenon links a country's economic development to its level of emissions. The Environmental Kuznets Curve (EKC) has gained popularity among scholars and practitioners as a concrete theoretical mechanism to model the relationship between economic development and environmental degradation (Grossman and Krueger, 1995, Pincheira and Zuñiga, 2021).[1] This theory suggests that as a country's economy grows, its environmental impact initially worsens. However, after reaching a certain level of development, the environmental conditions start to improve. Regarding CO_2 emissions, the EKC theory suggests that in the early stages of economic development, countries tend to emit more CO_2 as they rely heavily on fossil fuels for energy. However, as a country's economy continues to grow, it may transition towards cleaner and more sustainable forms of energy, such as green energy sources (solar, hydro, nuclear, etc.). As a result, the rate of CO_2 emissions may eventually decrease, potentially leading to a decline in overall greenhouse gas emissions. It is worth noting, however, that the existence of the EKC is a topic of debate among economists and environmentalists; where some argue that the relationship between economic growth and environmental degradation is more complex and context dependent.

To test some of the fundamental premises of the EKC, we compiled publicly available data from over 180 countries over a period of 30 years. We examined the country's GDP per-capita and CO_2 emissions within two five-year periods (1990-1995 and 2015-2019) to assess the trends in their emissions. Figure 1 illustrates some of the patterns that emerge from our data analysis.

[1] Based on the original work of Kuznets (1955) who studied the relationship between growth and income inequality.

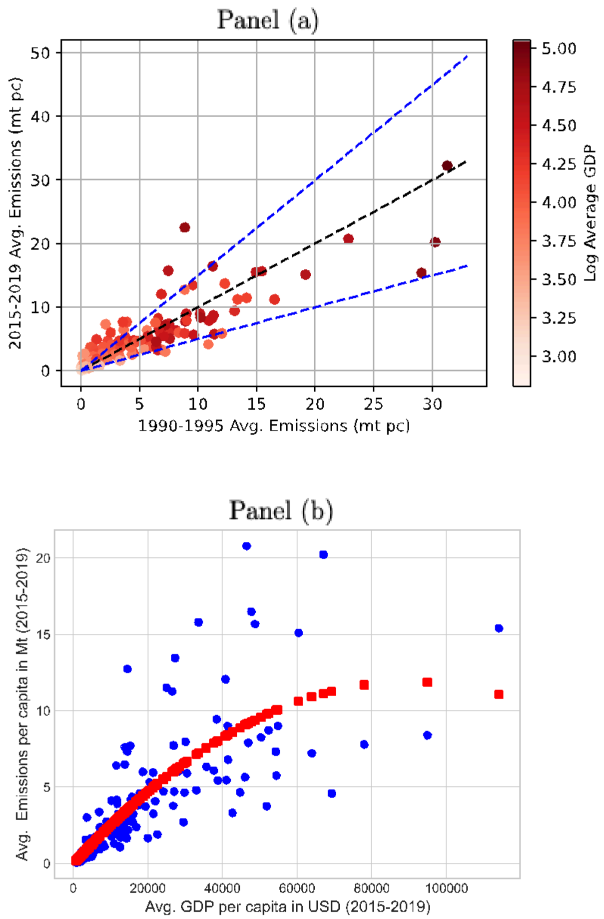

FIGURE 1: PANEL(A): Change in average CO_2 emissions per capita between the periods 1990-1995 and 2015-2019. Each point corresponds to a different country and the darkness of a dot reflects its average GDP per capita during the period 1990-2019 on a logarithmic scale, with darker colors representing higher GDP values. PANEL(B): Average CO_2 emissions per capita as a function of average GDP per capita. Dots correspond to the actual data while squares are estimates obtained fitting a quadratic regression.

In Figure 1, Panel (a), we plot the change in CO_2 emissions per-capita in metric tons (Mt) between 1990-1995 and 2015-2019, where each point corresponds to a different country and the darkness of a dot reflects its average GDP per-capita during the period 1990-2019 on a logarithmic scale (with darker colors representing higher GDP values). We have also drawn three lines to help visualize the changes. The continuous line is at the 45-degree angle while the two dashed lines have slopes 0.5 and 1.5. Thus, countries located close to the continuous line have experienced negligible change in their per-capita emissions over the last 25 years, countries that are close to the top dashed line have seen an increase around 50% on their per-capita CO_2 emissions and countries that are near the bottom dashed line have seen their per- capita emissions cut by about half. Roughly speaking, while we can see that some of the countries with higher GDP per-capita (darker dots) have seen a decrease in their emissions, the pattern is not at all uniform.

Panel (b) depicts the average CO_2 emissions (in Mt) per-capita as a function of GDP per-capita from 2015-2019. Dots correspond to the actual data while squares are estimates that we obtained by fitting a quadratic regression (using a higher degree polynomial regression result in overfitting):

$$\text{Avg. } CO_2 \text{ Emissions} = 2.655 \times 10^{-4}\,\text{GDP} - 1.473 \times 10^{-9}\,(\text{GDP})^2 \qquad \text{(EKC)}$$

with an R-squared of 0.602.

Based on the analysis, it seems that there is supporting evidence for the existence of an Environmental Kuznets Curve at a country level. However, the results of the quadratic regression model also suggest that CO_2 emissions tend to reach their peak when a country's GDP per-capita reaches around $90,000. Seeing as how only a handful of countries currently have a GDP per-capita above this threshold, this finding raises concerns. For reference, the average GDP per-capita across all countries in the world is approximately $11,371, while the average GDP per capita of the G7 countries (Canada, France, Germany, Italy, Japan, the United Kingdom, and the United States) is approximately $51,233, according to the World Bank.

Put together, the empirical evidence depicted in panels (a) and (b) suggests that relying solely on economic growth to improve environmental conditions may not be a sustainable approach –at least in the short-to-medium term– and that more comprehensive policies and initiatives will be necessary to address environmental issues.

Motivated by this need, this paper explores an alternative and complementary mechanism that can be used to accelerate the reduction in global CO_2 emissions, namely, the distribution of the population across urban areas. Specifically, we investigate how the distribution of the city sizes within a country affects its average level of CO_2 emissions. As we will demonstrate in the following sections, countries that can be classified as similar from an economic development standpoint exhibit different patterns of CO_2 emissions when analyzed through the lens of their population distribution. Moreover, our analysis reveals that in some cases, a country can reduce its emissions by promoting the concentration of its population in fewer, larger cities, while the opposite is true for other countries. The dynamic between city size distribution in a country and its CO_2 emissions is a complex, individualized, and fascinating perspective to research while attempting to drop emissions in the modern world.

2 A Model of CO2 Emissions based on City Sizes

In this section we present a structural mathematical model that connects the total annual CO_2 emissions of a country (or region) with two characteristics of its cities: the distribution of city sizes and the scalability of their emissions. Our proposed model builds upon two distinctive principles. First, is the celebrated Zipf's Law describing the power-law distribution of city sizes within a country. Also used is the urban scaling hypothesis, (Moran et al., 2018), which describes the relationship between city sizes and per capita emissions (Wei et al., 2021).

2.1 Zipf's Law

Named after the American linguist George K. Zipf, Zipf's Law states that the relationship between rank and frequency follows a power law distribution for many physical and societal phenomena. In his original work on the frequency of words used in a literary corpus, Zipf (1949) observed that in the English language there is a ratio of 2:1 between the two most used words, a ratio of 3:1 between the top and third most used words, a ratio of 4:1 between the top and fourth most used words, and so on. In other words, Zipf found that the frequency $f(r)$ of the r^{th} most used word in a literary corpus follows a power law distribution $f(r) = A/r^{\alpha}$ with $\alpha \approx 1$, where A is some normalizing constant (with $A \approx 0.1$ in the English language, Hosch, 2021).[2]

Since its introduction, Zipf's Law has been observed and studied in a variety of different contexts beyond linguistics ranging from biology and genetics (Furusawa and Kaneko, 2003) to economics (Gabaix, 2016) to city size distribution in a country (Gabaix, 1999, Ioannides and Overman, 2003, Düben and Krause, 2020), to name a few. Interestingly, a few decades before Zipf's work on the distribution of words on literary corpora, Auerbach (1913) had already observed that the distribution of city sizes (in terms of population) in a region follows a Pareto distribution of the form $N(S) = AS^{-\alpha}$, where $N(S)$ is the number of cities with a population of at least S individuals. Practically speaking, what Auerbach's observation tells us is that the population in a country is distributed among many small cities with only very few large ones that account for most of the population.

Motivated by the aforementioned existing literature on Zipf's Law and its connection to city size distribution, we model the tail distribution of city sizes in a given country using the power law

$$\overline{F}(S) = \left(\frac{S}{S_{min}}\right)^{-\alpha} \quad S_{min} \leq S \leq S_{max} \qquad \text{(Zipf's Law)}$$

[2] A few years later, Mandelbrot (1953) proposed the slight generalization $f(r) = A/(B + r)^{\alpha}$ (known as Zipf-Mandelbrot Law) to correct for some observed overestimation of the original Zipf's Law at low ranks.

where $\overline{F}(S)$ is the fraction of cities with a population greater than or equal to S. The shape parameter α characterizes the power law (which is typically close to one according to Zipf's Law, see Ioannides and Overman, 2003 for details) and S_{min} and S_{max} are country-specific parameters characterizing the minimum and maximum city sizes, respectively. The shape parameter α quantifies the skewness in the distribution of city sizes within a country. A small α indicates that most cities are relatively small, whereas a large α indicates that the majority of cities are relatively large. Also, for future references, we define $R := S_{max}/S_{min} \geq 1$, which is a normalized measure of the degree of dispersion of the distribution of city sizes within a country. Countries with R close to one tend to have cities of similar sizes while the opposite holds when R grows large.[3]

To illustrate the goodness-of-fit of Zipf's Law, Figure 2 depicts the logarithm of the city population versus the logarithm of the city rank for the largest 200 US cities (left panel) and largest 400 Chinese cities (right panel). As we can see, the fit of Zipf's law is exceptionally accurate, especially for the US data. The values of the shape parameter for the US and China are $\alpha_{US}=1.402$ and $\alpha_{China}=0.948$, respectively. For the purpose of comparison, the figure also shows the fit that we would have obtained if we were to assume that the distribution of city sizes within a country follows a Normal distribution. It is interesting that the popular Normal distribution performs rather poorly as a model to represent the distribution of city sizes within a country when compared to the power law distribution embedded in Zipf's Law.

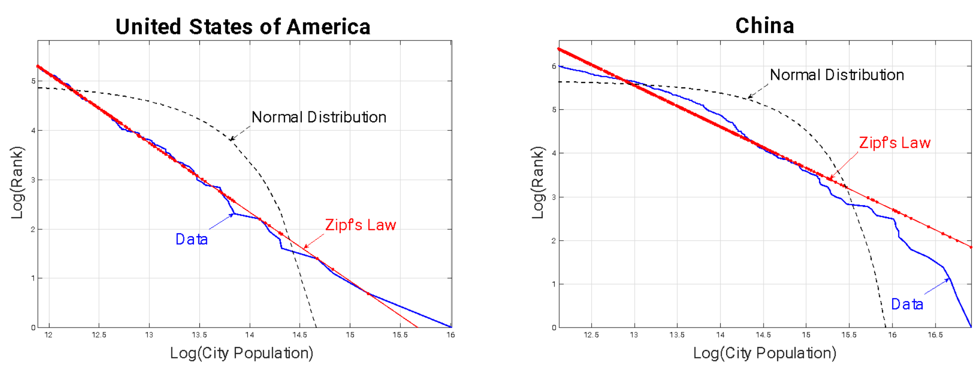

FIGURE 2: Log City Population versus Log Rank for top 200 US cities (top panel) and the top 400 Chinese cities (bottom panel).

[3] Capturing a country's city size dispersion using $R := S_{max}/S_{min}$ is advantageous for two reasons. First, it aligns mathematically with Zipf's Law, which is also defined on relative city sizes. Second, as a normalized measure, it is suitable for comparing countries with different population scales, unlike alternatives like standard deviation, which lack this normalization.

2.2 Urban Scaling Law

Let us now turn to the connection between per capita CO_2 emissions and population size of an urban area. By now, there is mounting empirical evidence pointing to a non-uniform distribution of per capita CO_2 emissions across city sizes (see for example, Moran et al., 2018, Seto et al., 2021, Wei et al., 2021 and references therein). Some of this evidence supports the Urban Scaling Hypothesis under which the per-capita emissions $\mathcal{E}_C$ of a city follows a power-law distribution as a function of its population size S (see for example Brown et al., 2009, Bettencourt and West, 2010, Ribeiro et al., 2019), that is,

$$\mathcal{E}_C(S) = A\, S^{-\beta}, \qquad \text{(Urban Scaling Law)}$$

for some scale parameter β and constant $A > 0$. The value of β controls the magnitude of any possible economy of scale in the level of CO_2 emissions of a city as a function of its population. For cities with $\beta > 0$ per-capita emissions decrease as the city size increases while the opposite holds true when $\beta < 0$. In the boundary case, $\beta = 0$, per-capita emissions are independent of the size of a city. Alternatively, β can be interpreted as the elasticity of per-capita emissions with respect to city size: a 1% increase in city size corresponds to a β% decrease in emissions.

Table 1 shows the values of the scale parameter β for ten countries using data from the *Global Gridded Model of Carbon Footprints* (see NTNU Industrial Ecology, 2022).

Country	β	**Country**	β
Canada	-0.1545	Russia	-0.0034
China	-0.0272	Saudi Arabia	-0.0926
Germany	-0.023	South Korea	0.0349
Japan	0.0724	UK	0.0066
Netherlands	0.0172	USA	-0.0236]

TABLE 1: Urban scale parameter for selected group of countries based on data from the Global Gridded Model of Carbon Footprints.

It is worth noticing that out of the four largest industrialized economies (US, China, Japan and Germany) three of them have a negative scale parameter. This suggests that many of the more industrialized countries could benefit from concentrating their population into a handful of major cities.

2.3 Urban Emission Scaling Factor

In this section we combine the specific characteristics of a country –captured by Zipf's Law and Urban Scaling Law– to express the relationship between CO_2 emissions and the distribution of its population across urban areas in terms of the country's scaling parameters. To this end, let us consider a hypothetical country with a total of N cities. From Zipf's Law, it follows that the number of cities with a population size between two values S and $S + dS$ is equal to $N[\bar{F}(S) - \bar{F}(S + dS)] = - N\bar{F}'(S)dS = N(\alpha/S)\,\bar{F}(S)dS$, where the last equality uses the power-law form of Zipf's Law. Thus, we can express the size of the country's population in terms of N as follows:

$$\text{Population} = \int_{S_{\min}}^{S_{\max}} S\,N\,\left(\frac{\alpha}{S}\right)\,\bar{F}(S)\,\mathrm{d}S = N\,\alpha\,\int_{S_{\min}}^{S_{\max}}\left(\frac{S}{S_{\min}}\right)^{-\alpha}\,\mathrm{d}S.$$

Using a similar derivation, we can combine Zipf's Law and Urban Scaling Law to express the total CO_2 emissions of the country as a function of N:

$$\text{Total Emissions} = \int_{S_{\min}}^{S_{\max}} \mathcal{E}_{\mathrm{C}}(S)\,S\,N\,\left(\frac{\alpha}{S}\right)\,\bar{F}(S)\,\mathrm{d}S = \frac{N\,\alpha\,A}{S_{\min}^{\beta}}\int_{S_{\min}}^{S_{\max}}\left(\frac{S}{S_{\min}}\right)^{-(\alpha+\beta)}\,\mathrm{d}S.$$

Combining the expressions above for the country's population and its total CO_2 emissions, we can compute the country's average CO_2 emissions per-capita. We summarize these steps in the following proposition. Recall that $R = S_{max}/S_{min}$.

Proposition 1. (Avg. CO_2 Emissions) *Consider a country (or region) whose city size distribution satisfies Zipf's Law and its annual per-capita emissions at the city level are governed by the Urban Scaling Law. Then, the country's average CO_2 emissions per-capita, $\mathcal{E}_T$, can take one of the four following values depending on α and β:*

$$\mathcal{E}_{\mathrm{T}}(\mathcal{R},\alpha,\beta) = \mathcal{E}_{\mathrm{C}}(S_{\min})\begin{cases} \frac{(1-\alpha)}{(1-(\alpha+\beta))}\left(\frac{\mathcal{R}^{1-(\alpha+\beta)}-1}{\mathcal{R}^{1-\alpha}-1}\right) & \text{if } \alpha\neq 1 \text{ and } \alpha+\beta\neq 1\\ (1-\alpha)\left(\frac{\ln(\mathcal{R})}{\mathcal{R}^{1-\alpha}-1}\right) & \text{if } \alpha\neq 1 \text{ and } \alpha+\beta = 1\\ \left(\frac{1-\mathcal{R}^{-\beta}}{\ln(\mathcal{R})}\right) & \text{if } \alpha = 1 \text{ and } \alpha+\beta\neq 1\\ 1 & \text{if } \alpha = 1 \text{ and } \alpha+\beta = 1. \end{cases} \tag{1}$$

Proof of Proposition 1: Integrating the expressions for the population and total emissions of a country we get

$$\text{Population} = N\,\alpha \int_{S_{\min}}^{S_{\max}} \left(\frac{S}{S_{\min}}\right)^{-\alpha} \mathrm{d}S = N\,\alpha\, S_{\min} \begin{cases} \frac{\mathcal{R}^{1-\alpha}-1}{1-\alpha} & \text{if } \alpha \neq 1 \\ \ln(\mathcal{R}) & \text{if } \alpha = 1 \end{cases}$$

and

$$\text{Total Emissions} = \frac{N\,A\,\alpha}{S_{\min}^{\beta}} \int_{S_{\min}}^{S_{\max}} \left(\frac{S}{S_{\min}}\right)^{-(\alpha+\beta)} \mathrm{d}S = \frac{N\,A\,\alpha}{S_{\min}^{\beta-1}} \begin{cases} \frac{\mathcal{R}^{1-(\alpha+\beta)}-1}{1-(\alpha+\beta)} & \text{if } \alpha+\beta \neq 1 \\ \ln(\mathcal{R}) & \text{if } \alpha+\beta = 1. \end{cases}$$

Dividing the total emissions by the population and noticing that $\mathcal{E}_C(S_{\min}) = A\,S_{min}^{-\beta}$ we get the expression for $\mathcal{E}_T$ in equation (1). □

The case α=*1* and *α*+*β* =1 in Proposition (1) implies that *β* =0. According to the Urban Scaling Law, in this case the population size of a city has no direct effect on the level of per-capita emissions. From a practical standpoint, countries with *β* =0 could not impact their emissions by adjusting the concentration of their populations across cities.

Note that the expression for the average per capita emissions of a country in (1) depends on the value of $\mathcal{E}_C(S_{\min})$. In order to have a benchmark measure that is independent of this country- specific variable and can be used to compare countries with different population sizes we introduce the following definition.

Definition 1. *The* Urban Emission Scaling Factor *of a country is defined by the ratio*

$$F := \frac{\mathcal{E}_T}{\bar{\mathcal{E}}_C} \qquad \text{(Urban Emission Scaling Factor)}$$

where $\bar{\mathcal{E}}_C$ *is the country's per-capita emission of its median-size city.*

Our choice to express a country's per-capita emissions relative to those of its median-size city is somewhat arbitrary and other normalizations are possible, for example by using the emissions of the largest or smaller city. However, these alternative choices appear less robust and could introduce additional measurement biases.

Corollary 1. *The urban emission scaling factor is given by*

$$\mathcal{F}(\mathcal{R},\alpha,\beta) = \begin{cases} 2^{\frac{\beta}{\alpha}} \frac{(1-\alpha)}{(1-(\alpha+\beta))} \left(\frac{\mathcal{R}^{1-(\alpha+\beta)}-1}{\mathcal{R}^{1-\alpha}-1}\right) & \text{if } \alpha \neq 1 \text{ and } \alpha+\beta \neq 1 \\ 2^{\frac{1-\alpha}{\alpha}} (1-\alpha) \left(\frac{\ln(\mathcal{R})}{\mathcal{R}^{1-\alpha}-1}\right) & \text{if } \alpha \neq 1 \text{ and } \alpha+\beta = 1 \\ 2^{\beta} \left(\frac{1-\mathcal{R}^{-\beta}}{\ln(\mathcal{R})}\right) & \text{if } \alpha = 1 \text{ and } \alpha+\beta \neq 1 \\ 1 & \text{if } \alpha = 1 \text{ and } \alpha+\beta = 1. \end{cases} \tag{2}$$

Proof of Corollary 1: The proof follows from combining the result in Proposition 1 and observing that the Urban Scaling Law implies the per-capita emissions of a country's median-sized city are equal to $\overline{\mathcal{E}}_{\mathrm{C}} = A\,(2^{\frac{1}{\alpha}}\,S_{\min})^{-\beta} = 2^{\frac{-\beta}{\alpha}}\,\mathcal{E}_{\mathrm{C}}(S_{\min})$. $\square$

Figure 3 depicts the value of $F(R, \alpha, \beta)$ using heatmaps in the (α, β) plane for four different values of R = 50, 100, 150, 200. A darker color indicates a smaller value of $F(R, \alpha, \beta)$, that is, lower levels of per capita emissions.

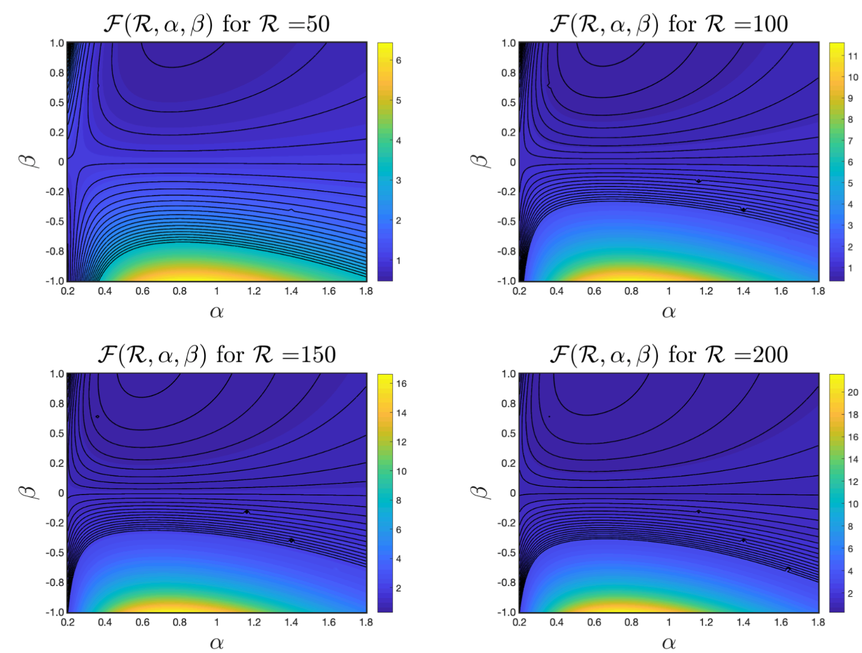

Figure 3: Heatmaps log of the emission scaling factor $F(R, \alpha, \beta)$ in the (α, β) plane for four different values of R =50,100,150,200.

Roughly speaking, the plots indicate that the value of the urban emission scaling factor $F(R, \alpha, \beta)$ is larger when β is negative and α is close to one. Also, its value increases as R increases. For example, in the case of the USA $(R, \alpha, \beta) = (90, 1.402, 0.0236)$ which results in an emission scaling factor $F_{US} = 1.027$. In the case of China, $(R, \alpha, \beta) = (180, 0.948, 0.0272)$ with a corresponding emission scaling factor $F_{China} = 1.056$.

To further assess the dependence of F on R, the following corollary of Proposition 1 looks at the extreme case $R{\downarrow}1$ and $R\to\infty$.

Corollary 2. *Suppose* $\alpha \neq 1$ *and* $\alpha + \beta \neq 1$*, then*

$$\lim_{\mathcal{R}\downarrow 1} \mathcal{F}(\mathcal{R},\alpha,\beta) = 2^{\frac{\beta}{\alpha}} \quad \text{and} \quad \lim_{\mathcal{R}\to\infty} \mathcal{F}(\mathcal{R},\alpha,\beta) = \begin{cases} 0 & \text{if } 0 < \alpha < 1 \text{ and } \beta > 0 \\ 2^{\frac{\beta}{\alpha}} \frac{(1-\alpha)}{(1-(\alpha+\beta))} & \text{if } 1 < \alpha \text{ and } 1-\alpha < \beta \\ \infty & \text{otherwise.} \end{cases}$$

Figure 4 depicts the value of $F(R, \alpha, \beta)$ in the limiting case $R = \infty$ in the (α, β) plane.

It is interesting to notice that the limiting behavior of F exhibits a singularity in the region when $\alpha \in (0, 1)$ and $\beta = 0$ where the urban emission scaling factor F jumps from $F = 0$ to $F = \infty$ as β goes from a positive value to a negative value.

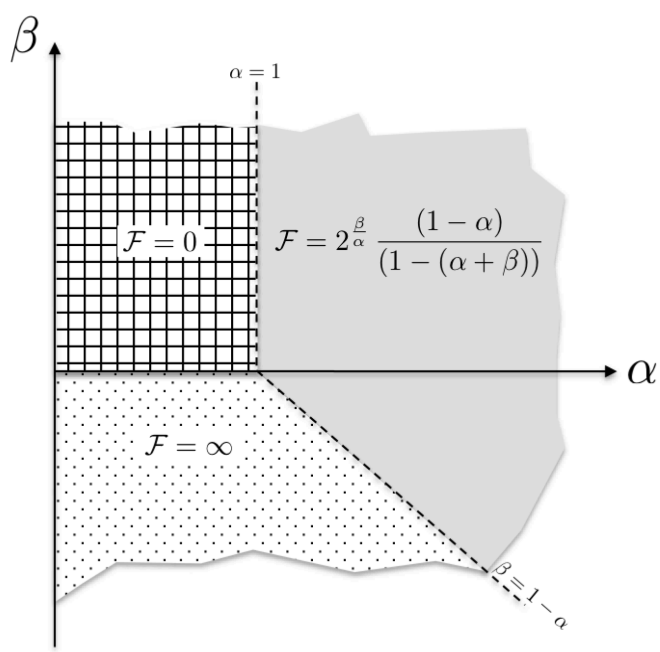

FIGURE 4: Limiting behavior of the urban emission scaling factor F in the limit as $R\to\infty$.

3 Practical Implications of the Urban Emission Scaling Factor

In the previous section we proposed a mathematical model to represent a country's total CO_2 emissions per-capita based on specific characteristics of the distribution of its population across cities and the pattern of CO_2 emissions at the city level. The resulting urban emission scaling factor F derived in Corollary 1 provides a concrete measure (an index) that summarizes the effect of these urban factors on a country's per-capita emissions.

In Table 2 we compute the value of the urban scaling factor for various countries combining available data from the World Bank and the *Global Gridded Model of Carbon Footprints* project.

Country	α	β	R	F
Canada	0.9841	-0.1545	34	1.194
China	0.8401	-0.0272	160	1.058
Germany	1.1219	-0.0230	29	1.022
Japan	1.3194	0.0724	101	0.918
Netherlands	1.3011	0.0172	9	0.992
Russia	1.2342	-0.0034	114	1.005
Saudi Arabia	0.8786	-0.0926	47	1.132
South Korea	0.9687	0.0349	152	0.938
UK	1.0560	0.0066	74	0.991
USA	1.0300	-0.0236	126	1.041

TABLE 2: Urban scale parameter for a selected group of countries based on data from the World Bank and the Global Gridded Model of Carbon Footprints.

As we can see the values of F range from a minimum value of 0.918 for Japan to a maximum value of 1.194 for Canada. Recall that countries with an F value above 1 have per-capita emissions that are greater than their median city. From a practical standpoint, this indicates that given their urban characteristics, they would benefit from a more uniform concentration of their population across metropolitan areas. Thus, countries such as the USA and Germany would see a greater decrease in emissions if they move towards developing fewer, bigger cities. On the other hand, countries with an F value below 1 have country-level per-capita emissions that are lower than the median value for their cities. In such cases, their emissions would decrease if their cities were more dispersed and the population more spread out. This includes countries such as Japan, The Netherlands, South Korea and the UK.

We can take this insight one step further by considering a hypothetical scenario in which countries with $F > 1$ could move towards a perfectly uniformly distributed population across cities (i.e., by letting R decrease to one) and countries with $F < 1$ move towards a perfectly spread out population across cities (i.e., by letting R grow boundlessly). In this idealized world we can define by F_{Ideal} the resulting minimum possible urban scaling factor that a country can achieve as well as the ratio

$$\Delta F := \frac{F_{Ideal}}{F} \qquad \text{(Emissions Reduction Potential)}$$

which captures a country's emission reduction potential relative to its current situation. Table 3 shows the values of ΔF for the same set of countries in Table 2.

Country	ΔF	**Country**	ΔF
Canada	75.10%	Russia	99.34%
China	92.40%	Saudi Arabia	82.09%
Germany	96.42%	South Korea	0.00%
Japan	92.22%	UK	90.69%
Netherlands	96.22%	USA	94.53%

TABLE 3: Emissions' reduction potential for a selected group of countries.

As we can see, the value of ΔF varies significantly from country to country. For instance, Canada's ΔF = 75.10%, indicating that the country has the potential to reduce its per-capita CO_2 emissions by about 25% if it could move toward a fully concentrated population. On the other hand, emissions in Russia would also decrease if the country's population gets more uniformly distributed, but in this case per-capita CO_2 emissions would decrease by less than 1%. Interestingly, for South Korea $\Delta F = 0$ which suggests that the country could hypothetically reduce completely its CO_2 emissions by further spreading about its population[4]. More research would have to be done in order to better understand the unique properties of countries with small values of ΔF.

4 Concluding Remarks

[4] Note that according to Table 2 for South Korean $\alpha < 1$ and $\beta > 0$ and so according to Corollary 2 its urban scaling factor F converges to zero as $R \to \infty$.

In this paper, we delved into the relationship between a country's CO_2 emissions and the distribution of its population across cities. In particular, we proposed a specific measure termed the 'Urban Scaling Factor', which consolidates two distinctive characteristics of a country and was captured through Zipf's Law and the Urban Scaling Law. This factor could serve as an index that can be utilized to evaluate how potential shifts in a country's population distribution could influence its CO_2 emissions.

Our initial analysis of the Urban Scaling Factor for the ten countries with publicly available necessary data (as listed in Table 2) indicates a considerable potential for reducing CO_2 emissions through a possible redistribution of population across cities (refer to Table 3). However, the outcomes are not universally consistent across our sample of countries. In certain cases, the prospective impact of population redistribution seems minimal.

Due to data limitations, the majority of our analysis concerning the practicality of the Urban Scaling Law was conducted for developed nations. In future research, we would like to broaden our analysis to encompass a greater number of countries, particularly those within the developing world, where data sufficiency is currently an issue. Furthermore, more research and investigation into how other factors (literacy rates, trade, or cultural values for example) all play a role in a country's CO_2 output. Climate change is a complex and multifaceted issue, as such there will always be more to study and understand.

In conclusion, through a combined usage of Zipf's law and the Urban Scaling Law, we find a potential mechanism to describe a country's emissions through the Urban Emission Scaling Factor. With this factor in mind, we can find one of two outcomes for how a given country can improve its emission rates: either through an increase in urbanization, or an expansion/distribution of the population. In the end, while climate change will require systematic and efficient action, a mindset of changing how countries distribute their populations can and will positively impact the growing push to save our planet.

Acknowledgement: The author is very grateful to Dr..Jorge Laval of The Georgia Institute of Technology for his invaluable guidance and continuous support, and to Dr. Kenny Fournillier of U-Chicago's Laboratory Schools for his careful reading of the paper and constructive comments. The author would also like to thank the journal editor, Kyra Hong, and the review team for their insightful feedback on a preliminary version of this paper.

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