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The Algebra of Empathy: How Timed Tests Are Inhibiting Theory of Mind in American Math Classrooms

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Abstract

Over the last century, academics in an expansive array of fields—from philosophy to art to neuroscience—have been arguing over the meaning of a certain moral objective: empathy. Although their definitions vary, they converge at a single idea. In education, empathy requires the ability to understand another being's cognitive state, a skill also referred to as theory of mind. In the context of the American math classroom, theory of mind is the sum of three relationships: student and teacher, student and peer, and student and self. Using research from across disciplines, this paper argues that the overall lack of empathy in the math classroom is halting students' theory of mind, and without that ability to perspective-take, primary school children are not mastering the essential quantitative topics necessary to succeed in the 2st century's rapidly changing economy.

Introduction

There are two types of people in the world: those who are good at math and those who are not. Americans apparently fall into the latter category. The 2015 Program for International Student Assessment (PISA) offers the latest evidence that the United States—the world's dominant economic and military power—is dramatically underperforming its peers. The most recent PISA results placed the U.S. at an unremarkable 38th out of 71 countries in math abilities. And among the 35 members of the Organization for Economic Cooperation and Development, the U.S. ranked 30th (Kerr, 2016). When it comes to math in America, something does not add up.

Across the globe, the lowest achievers were math memorizers, students who think of the subject as a list of memorized steps rather than an interconnected web of big ideas, and the U.S. contained the highest numbers of them (Boaler & Zoido, 2016). So why is a powerful and wealthy country—with enough resources to become the star of international education—failing to produce deep mathematical thinkers? American teachers often teach math as a series of rules to memorize. You learn a pattern and implement it numerous times in methodical worksheets. Next, you apply those math facts to similar problems but under high-stress timed conditions—to test your proficiency. This system is driving mathematicians, educators, and even neurobiologists to consider how timed exams and pattern learning might influence students' mathematical comprehension and shape classroom relationships.

Within months of the troubling 2015 PISA results, a key term surfaced in education research: empathy, or as University of Toronto's Tyler Cosante writes in *Edutopia*, "the intrapersonal realization of another's plight that illuminates the potential consequences of one's own actions on the lives of others" (as cited in Hollingsworth, 2003, p. 146). To educators like Cosante, empathy is understanding a social world. It cultivates theory of mind, the capacity to imagine the cognitive states of other people. In the math classroom, empathy is inspecting an equation using multiple methods, rather than regurgitating memorized steps. It is appreciating how someone else would think about the very same problem and discussing how different students' approaches overlap or diverge. This ability to collaborate, perspective-take, and thus empathize is lacking in the typical U.S. Algebra class. On the other hand, timed tests, high pressure exams that require exercises focusing on an individual's performance, remain abundant. This contrast suggests an inverse relationship between timed tests and empathy in the math classroom.

Though many scholars condemn the use of timed assessments, arguing that they goad competition, reduce students to numbers, and feed into math anxiety, they do not address the effect of timed math tests on cognitive perspective taking. Through research in the fields of education, mathematics, neuroscience, psychology, and philosophy—as well as reflection on my personal math experience—I have attempted to close this gap. My conclusion is simple: timed tests are inhibiting empathy in American math classrooms. In order to support this claim, I will outline how timed tests are corroding three sets of relationships within the math classroom: student and teacher, student and peers, and student and self. Next, I will argue why the accumulation of these harmed social interactions is creating an empathy deficit, which, in turn, is stunting American math students' and teachers' theory of mind. In detailing this domino effect, I will provide a solution to the flaw in our current U.S. math equation.

Student and Teacher: Categorizer or Catalyst?

Let us rewind to my second grade math classroom. It is April 2007. "Ok children. It's time to test our multiplication tables!" Ms. Riley chirps. The kids squirm. She passes out the sheets of problems and starts a timer. I grip onto my pencil. Six times four. Six times five. Six times six. Six times seven… Ms. Riley collects the papers and sorts through them as we munch on our morning snack. Then she divides us into two groups. Although the groups are called "Blue" and "Red," it is clear—even to seven and eight-year-olds—that blue means "good math student" and red means "bad math student."

This memory is a familiar one. Across the United States, math teachers employ timed multiplication tests to assess and sort students. But can a system that drives teachers to instantly judge their students diminish empathetic relationships?

Most educators and psychologists concur that empathy is essential in a student/teacher relationship, but some disagree about whether timed tests disturb that very same connection. According to teachers Brianna Crowley and Barry Saide, empathy is the act of projecting oneself into an observed context (Crowley & Saide, 2016). It allows teachers to recognize how to help students, generating greater critical thinking. Crowley also notes that empathy increases the child's exposure to diverse techniques and ways of approaching a single problem, which facilitates communication between mentor and student. Roger Antonsen, a mathematician from the University of Oslo, agrees. In his TED talk *Math is the Secret to Understanding the World,* he states that as a teacher, "If I tell a story from a different point of view, I enable [deeper] understanding" (Antonsen, 2015). To Antonsen, empathy is a catalyst. If teachers cultivate empathy, they will develop stronger relationships with their students, who will become more open-minded in return. Empathy also enhances understanding and retention by allowing students to feel as though they are connected to the stories and topics presented by their teachers.

Although these educational theories are encouraging, they are not prevalent in many U.S. math classes. Crowley and Saide argue that the majority of math educators "are teaching content instead of students" (Crowley & Saide, 2016). American education highlights the essential role of reason and logic but tends to pay less attention to the equally critical role that emotional aptitudes play in propagating success. That is the main problem with Ms. Riley's timed multiplication tables. High stakes math tests starting as early as first grade force teachers to immediately categorize students. If you perform quickly, teachers—often unconsciously—send the message to students that they are "good" at math. If you perform slowly, you just are not a math person.

But according to Cathy Seeley and Jo Boaler, people are not simply "good" or "bad" at math, like I suggested in the opening of this paper. Maybe Americans are not "bad" at math; they just do not know how to teach it. Seeley, a mathematics educator and writer in Texas, and Boaler, a professor of mathematics education from Stanford University, explore the dangers of the good at math/bad at math dichotomy. In her book *Faster Isn't Smarter*, Seeley argues that our culture erroneously equates fluency with timed testing. She states, "We must look for ways to tap into each student's strengths, not fall into the trap of believing that timed tests are the way we identify our good mathematics students" (Seeley, 2015). In this call to action, Seeley warns that we too easily fall prey to a categorization mentality. Boaler similarly condemns the use of timed tests to assess math facts and facility. She explains how teachers in the U.S. are obliged to follow directions that make little sense to them and do not coincide with research evidence. A quote from a first-grader sums up this sentiment. One young boy complained to Boaler that there was "too much answer time and not enough learning time" in his school (Boaler & Zoido, 2016). The current math classroom is a performance-driven mill, where a teacher's main purpose is "to order and categorize students" (Boaler, 2016). Crawley, Saide, Seeley, and Boaler all converge on the same idea. With timed tests, the relationship between teacher and student becomes mechanical instead of personal.

These examples all paint timed tests as purely negative, but are there any benefits to the current system? According to *Pearson's Education Assessment Report,* there are. The three authors of this report argue that timed tests are necessary in identifying students who are struggling and those who have great math potential. They offer a capitalist argument: "Standardized testing procedures serve the same function in education as they do when applied to goods and services encountered every day. Without standardized weights and measures, things as simple as purchasing fresh produce or a gallon of gasoline would be quite chaotic" (Brooks et al, 2003). In other words, timed testing is just like buying apples at Trader Joe's. You need certain measures and tests, like prices, to separate the best fruit from the rotten ones. *Pearson's Education Assessment Report* states that categorizing students is helpful, rather than harmful. With standardized timed tests, teachers can easily see who is struggling and give those students the resources to improve. Timed tests clearly have some value in identifying students' levels of understanding. However, their increasing importance in American math curricula may be overriding any benefits. When we expand the discussion to other classroom relationships, we can see that these tests are diminishing empathy.

Student and Other Student: Friends or Foes?

Jump back to Spring 2011. I am now in sixth grade. It is the day of ERB tests, and a buzz about them vibrates through my sixth grade Advanced Math classroom. ERB tests—timed, multiple choice assessments for students in independent and public schools within and outside of the United States—are a yearly tradition at my school.

"I solved that really hard problem about bees," shouted one boy.

"Ah man. I didn't. But I totally aced the one after it. I am sure the answer was 25," exclaimed another.

"Me too!"

"Same here."

"It was so obvious. I can't believe people couldn't solve it!" screamed a third. I glance about the room. Some students—mostly boys—are jumping up and down, comparing answers. Other sit quietly, heads down.

Although my small independent and international school does not administer as many standardized tests as other school systems, ERB week frequently turns civil students into stressed-out rivals. And the level of emphasis on these assessments is growing exponentially. According to a two-year study of 66 school systems across the United States, kids are mandated to complete too many standardized tests (Hart et al., 2015). Additionally, there is no evidence revealing that additional testing time improves student achievement. Instead, these tests raise competition among peers. The average student in America's big-city public schools takes roughly 112 mandatory standardized tests between pre-kindergarten and the end of twelfth grade — an average of about eight tests a year. That dissolves an annual 20 to 25 hours of teaching time (Hart et al., 2015). This research connects back to the quote from Boaler's first grader. Students are spending too much time testing math facts ("answering time") and not enough time grasping the material (i.e., "learning time"). In consequence, this system breeds performance-driven, fixed-mindset students.

Mindset, a term coined by Stanford University psychologist Carol Dweck, is the belief about one's basic qualities. A fixed mindset is the belief that intelligence and talents are innate and immovable. A growth mindset, on the other hand, is the belief that intelligence and other basic qualities are supple and can be developed through dedication and hard work (Dweck, 2008). People are not born with a fixed or growth mindset. Dweck and other psychologists repeatedly demonstrate that environments, like schools, are the main agents behind these attitudes. Competitive environments, often ones that highlight performance (such as math tests), develop fixed-mindset children. In these environments, one student's gain is another's loss, so students may withhold effort to avoid being stigmatized as a "teacher's pet" or belittle peers' achievements (Dweck et al, 2014). In fact, in a study that explored the psychological mechanisms that allow adolescents to thrive in mathematics in junior high, Dweck and Blackwell made a surprising discovery. Even when students on both ends of the mindset continuum showed equal intellectual ability, their beliefs shaped their responses to academic challenge. In a questionnaire that measured motivational variables, Dweck and Blackwell found that students with predominantly fixed mindsets were more likely to cheat on a test (Blackwell et al, 2007). Cheating becomes more likely when the number of problems you answer correctly within a set amount of time is what defines you.

Dweck's research suggests that competitive environments are associated not only with lower achievement and negative strategies, but also with students liking each other less (Dweck et al., 2014). Other psychologists support the idea that timed tests are ruining peer relationships. In *A Matter Of Time: Emotional Responses To Timed Mathematics Tests,* Walen and Williams (2002) explain how in a study of prospective elementary school teachers, they found that a significant number of children experienced their first "traumatic encounter" with mathematics as early as in grades three and four. Taking timed tests in competition with peers was the main contributing factor (Walen & Williams, 2002). In addition, classrooms that encourage competition and individualistic goals are particularly ill suited to minority students, who are more likely to be raised in interdependent cultures (Dweck, 2008).

Here is why. According to *The Journal of Economic Behavior and Organization*, timed tests spawn a devastating consequence —

the "stereotype threat." If a student is worried about confirming negative perceptions of a group to which she is a member, it can hurt her individual performance. Take a look at women in math. In the U.S., the majority of people believe that males have a stronger number sense. Three researchers tested that hypothesis by running a series of math contests in elementary schools. Subjects competed in a sequence of up to five contests. Males held a significant advantage during the first contest, but when the competition lasted for three rounds, girls began to outscore boys. What's more, the first-round advantage for boys evaporated if the time element was removed from that competition (Cotton et al., 2013). Males aren't inherently better at math. They just react more favorably than females (at least in the short term) to the competitive incentive of timed tests. But the misconception leads to anxiety-ridden girls, falsely confident boys, and an overall competitive atmosphere. The results of this study also raise questions about whether the male competitive advantage might drive long-run achievement differences between the sexes. If females believe from an early age that they are weaker math students, they are less likely to pursue STEM fields, strengthening the gender divide in these areas (Dweck, 2008).

Boaler calls this competitive incentive "one of the most damaging myths that pervades U.S. math classrooms" (Boaler, 2016). It leads to the categorization of people with or without a "math brain." And with only timed tests to measure them, children who are faster remain faster while children who are slower remain slower, increasing the gap between these two groups. Even more harrowing, competition and stereotype threat garner a serious mental consequence, which "affects about 50 percent of the U.S. population and more women [and minorities] than men" (Boaler, 2016). It's called math anxiety.

Student and Self: Me or My Anxiety?

Now let's travel back to June 2, 2016. I'm in eleventh grade, taking my International Baccalaureate Math Higher final exam. Mr. O instructs us to begin. My heart is pounding. I open the page and begin the first problem. I realize I have no idea how to solve it. My heart rate is now skyrocketing. I feel light-headed. I flip to the next problem. Despite my weeks of preparation, I cannot do it. Instead, my brain reverberates with: "I knew I shouldn't have taken this challenging math class." "Everyone else in this class is so much smarter than me." "How am I going to get into college?"

Unfortunately, this situation is a common one. Nearly six out of ten university students suffer from some form of math anxiety, and this phenomenon affects kids as young as seven years old (University of Granada, 2009). Three neurobiologists tested 46 seven to nine-year-old second and third graders. They used the Scale for Early Mathematics Anxiety (SEMA), a standardized method for assessing math anxiety in older children and adults, to measure the students' stress levels. This test divided participants into high-math-anxiety (HMA) and low-math-anxiety (LMA) groups. From there, the researchers asked the children to determine whether addition and subtraction problems were correct (e.g., "2 + 5 = 7") or incorrect (e.g., "2 + 4 = 7"). While the students were solving these problems, fMRI imaging detected their brain activity. The results were shocking. The neurobiologists found that students with high math anxiety exhibited hyperactivity and abnormal effective connectivity of the amygdala, a brain region in charge of processing negative emotions and fearful stimuli. More striking, the scans of these students depicted reduced responses in cortical and subcortical areas, parts of the brain that are crucial for mathematical and numerical reasoning in both children and adults. When trying to solve math problems under pressure, these kids were able to process fear more easily than numbers. At the end of this paper, the neurobiologists suggest that we need to "spur new ways of thinking" about math education (Young et al, 2012). Our current system of memorizing mathematical facts and applying them as quickly as possible in a timed scenario is not just lowering scores. It is altering our brains' basic functioning.

In Jo Boaler's work, she cites this study, as well as others, to show the significant effect this issue is having on our society. Timed tests are the stimulus of the early onset of math anxiety. Worst of all, time pressure blocks working memory, a system for temporarily storing and managing the information required to carry out complex cognitive tasks and therefore hinders students' abilities to problem-solve. Math anxiety influences even those with *high* amounts of working memory, "precisely those students who have the greatest potential to take mathematics to high levels" (Boaler, 2016). Seeley expands on this point by explaining how some of the world's greatest thinkers, scientists, and mathematicians have never been fast at arithmetic, even though they were tremendously successful in working with higher-level mathematics. These slow mathematicians include Maryam Mirzakhani, the only woman to win a Fields Medal, Michael Faraday, the inventor of the electric motor, and even Charles Darwin. Good math is not fast math. Measuring one aspect of mathematics (fast recall) is "flawed as an assessment approach" (Seeley, 2015). It harms students' confidence and desire to solve new problems, and it exacerbates anxiety, resulting in math avoidance and an overall negative experience.

Sharon Walen and Steven Williams, two Professors of Mathematics, agree. Through the stories of two adult women and one grade three girl, these researchers illustrate both the immediate and the long-term effects of timed skills testing in mathematics. They argue that the negative emotional responses expressed by their subjects were not directed towards mathematics, nor even towards assessment, but specifically towards the timed nature of the tests they were required to take: "It seems particularly foreign for teachers to value the time that one takes to complete a problem over the quality of their students' understanding or solutions" (Walen & Williams, 2002). Like Boaler and Seeley, these researchers argue that math memorization and timed testing are particularly problematic in the face of today's challenge for students to think and communicate mathematically. In cases when mathematics is used in the real world, speed is rarely a primary concern, so why is it emphasized so much in today's classrooms? (Walen & Williams, 2002). Until recently, many jobs did not require math beyond simple arithmetic. But in today's technologically driven era, advanced quantitative skills are becoming essential. The combination of rapidly increasing cases of math-traumatized students and the demand for mathematically literate workers make empathy-driven education a necessity. Americans may not be inherently crummy mathematicians, but their system of teaching is falling apart. How can teachers teach Algebra to a class full of terrified children? Simply put, math education needs a new solution.

Changing Minds

From where we left off in my math narrative, it seems as though I despised my IB Higher Level Math class. That is not the case. I nearly failed that exam but became aware of a calamitous attitude. Up until that moment, I was a math memorizer, just like the majority of American PISA participants. I could easily retain math facts and expel them in certain timed assessments. That method was successful, until suddenly it was not. Over the years, as the little first grader would exclaim, I was acing the "answer time" and failing the "learning time." Throughout my senior year, I paused and reworked how I was approaching the subject. I reframed my beliefs about my own abilities and began viewing my classmates as helpers rather than competitors who could raise the test curve. I spent hours in extra sessions with my teacher, delving deep into the roots of problems, rather than skimming the math facts. In all, I survived. Even better, I began to enjoy the epiphany of understanding a complex calculus or statistics problem and marveled at how mathematical concepts linked to everyday life. By building up these sets of relationships, not only did I strengthen my math capabilities, but also I developed something else: empathy.

Roger Antonsen, the mathematician mentioned earlier in this paper, introduces an interesting and related argument. He asserts that there is a domino effect between math and empathy. Empathy in the classroom allows for students to better understand and appreciate math, which allows them to become more empathetic people. Antonsen explains:

> Mathematics and computer science are the most imaginative art forms ever…...When I view the world from your perspective, I have empathy with you. If I really, truly understand what the world looks like from your perspective, I am empathetic. That requires imagination. And that is how we obtain understanding. And this is all over mathematics and this is all over computer science, and there's a really deep connection between empathy and these sciences (Antonsen, 2015)

Throughout the day, we find patterns and represent them. In order to fully comprehend these problems, we have to change angles (figuratively and literally). Take the equation $x + x = 2 \cdot x$. This is a true pattern, because $5 + 5 = 2 \cdot 5$. But think about it. One expression is equal to another, and that's two different perspectives. One perspective is a sum, a series of symbols added together. On the other side, it is a multiplication. Antonsen goes as far as to say that every equation is like this—every mathematical equation where you use that equality sign is a metaphor. It is an analogy between two entities. You are viewing something, grasping its two different points of view, and expressing that in a language (Antonsen, 2015). In short, by doing math, you are taking another perspective, appreciating it, and grasping it. I call this the Algebra of Empathy.

Empathy is a moral and social goal. Philosophers, educators, psychologists, and everyday people have and are constantly arguing over its definition. At the heart of empathy is theory of mind, or the ability to interpret another perspective. In the context of the math classroom, theory of mind is the accumulation of the three social interactions I have discussed throughout this paper: student and teacher, student and peer, and student and self. It looks like this:

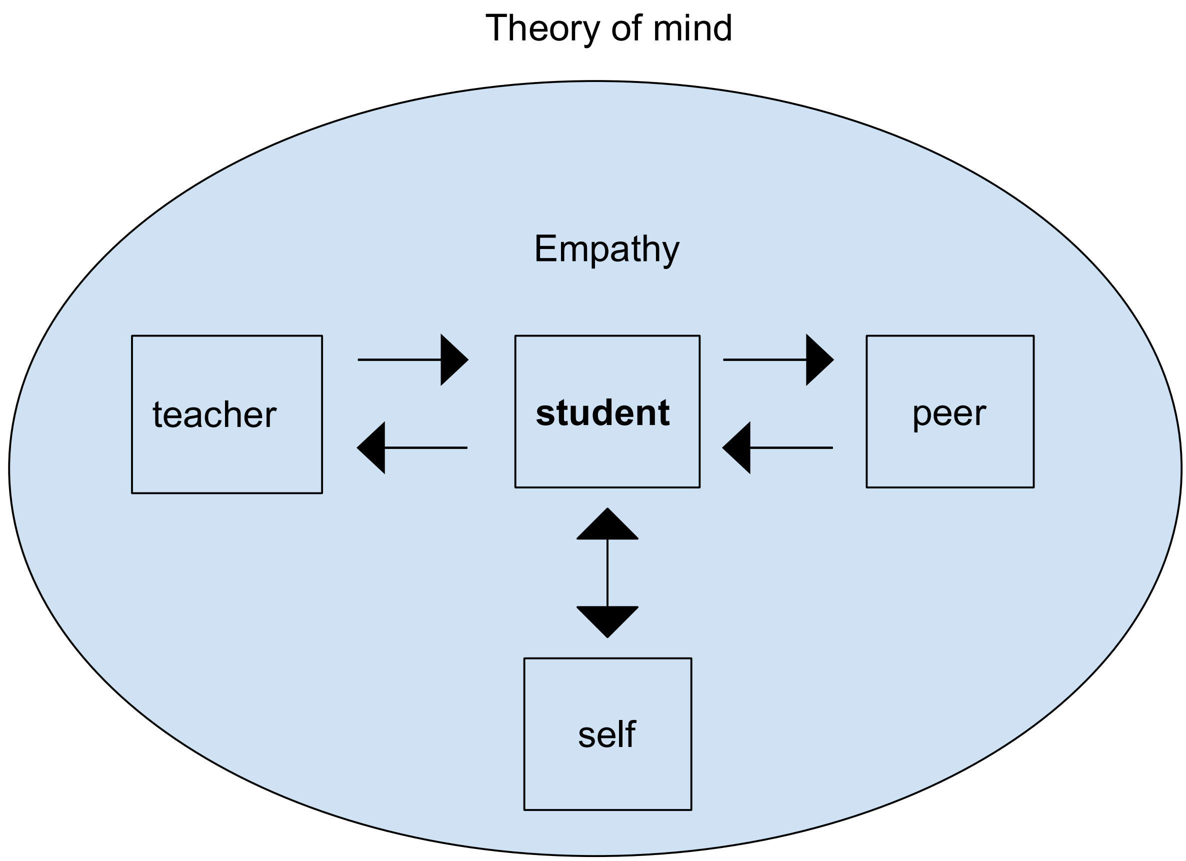

FIGURE 1: Map of relationships in the math classroom, titled *Theory of Mind*

Math is about relationships between concepts and people. Relationships require theory of mind, which is fueled by empathy. The current math education system in the United States is lacking the understanding of the importance of relationships, and this void could be reflected in the low math scores relative to other countries. As I have argued throughout this paper, timed tests are a major component of this downward spiral. By fixating on the performance of kids under timed settings, teachers are inclined to categorize students, who then feel judged in return. Raising test scores on a pedestal encourages peers to compete with each other rather than help one another. Finally, this high pressure atmosphere fosters math anxiety, marring students' feelings about themselves. The overall lack of empathy is stunting our theory of mind, and without that ability to perspective-take, kids are not mastering essential quantitative topics.

To create a solution, we could begin by recuperating these relationships. We could start with a bottom-up approach of reducing the number of timed tests in K through 12 math education. By emphasizing the human, emotional aspects of mathematics and presenting the subject as "one of open, visual, creative inquiry," we could repair the three essential social interactions in the math classroom (Boaler & Zaido, 2016). It is as straightforward as pausing. It is Ms. Riley stopping to ask her second grade students if they understood the significance of multiplication rather than immediately sorting them into "Red" and "Blue" groups. It is making mandatory standardized tests less of an importance in school culture. It is Mr. O walking me through an integral proof right after my disastrous exam, explaining the connections between the two ways of approaching the problem. This sense of empathetic communication, if implemented in schools across America, would facilitate theory of mind, which could then lead to a comprehension of mathematics far beyond memorizing multiplication tables or the steps of a proof. And then maybe, through this new type of Algebra, the United States will not be so bad at math after all.

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